Abstract

We extend the well-known form closure framework for rigid parts to holding a class of deformable parts. We consider 2D deformable parts modeled as polygons with a triangular FEM mesh and given stiffness matrix that models linear elasticity and the potential energy associated with deformations of the mesh. We define the D-space (deformation-space) of a part E as the C-space of all nodes in the FEM mesh M representing E. The free space $D_{\text{free}}$ is defined as the intersection of the set of topology preserving mesh configurations with the complement of the union of all D-obstacles that represent collisions of the part with finger bodies. 

Given mesh M of part E, FEM stiffness matrix K, and A, the set of finger bodies in frictionless point contact with E, $U_A$ is the minimum work required to release the part. We say A holds E in deform closure if $U_A > 0$. We present a numerical example and prove two results: (1) If contact set A holds a rigid part in form closure, A will hold the equivalent deformable part in deform closure and (2) Deform closure is frame invariant.

1. Introduction

There is a substantial body of research on robotic holding (grasping and fixturing) of rigid objects. In this paper we consider holding deformable parts. Unlike rigid parts, deformable parts cannot be fixed in a single configuration. We extend the C-space concept of form closure by defining deform closure.

In C-Space, the part’s position and orientation are parameterized in a Euclidian space thus representing the part’s configuration in physical space as a point in C-Space [Selig00, Mason01]. The set of points in C-Space for which the corresponding configuration of the part intersects with a given obstacle is the corresponding C-obstacle. When a part is in contact with an obstacle, its image in C-Space lies on the C-obstacle’s surface. The complement of the union of all C-obstacles where the part is allowed to move freely is called the Free Space ($C_{\text{free}}$) of the part. Immobility is achieved at a configuration $q_0$ if no neighboring configurations in C-Space lie in $C_{\text{free}}$.

We define the D-space (deformation-space) of an elastically deformable part E as the C-space (configuration space) of all nodes in the FEM mesh M representing E. This FEM model of the part causes the perimeter to behave like a polygon with variable length edges, with hinges at each vertex. Given the initial configuration of the mesh $q_0$ in D-space, we define $D_T$ the set of configurations that possess the same mesh topology as $q_0$. We consider frictionless contact of E with a set of rigid finger bodies $A = \{ A_i, \ldots, A_k \}$. These obstacles are represented as D-obstacles $D_A$, that are the configurations that result in a collision between the mesh and A. The free space $D_{\text{free}}$ is the set of “allowed” configurations, defined as the intersection of $D_T$ with the complement of the union of D-obstacles.

2. Related Work

Bicchi and Kumar provide a concise survey of literature on grasping and fixturing in [Bicchi00]. Grasps of rigid bodies can be classified as force or form closure. Form closure or immobility occurs when any neighboring configuration of the part results in collision with an obstacle. Force-closure occurs if any external wrench can be resisted by applying suitable forces at the contacts [Mason01, Rimon98]. Gripper contacts can be modeled as frictional points, frictionless points or soft contacts [Salisbury82].

The mobility of rigid bodies in contact with frictionless finger bodies was initially studied using first order approximations [Realeaux1876, Somoff1900, Mishra87, Markenoff90]. The first order theories are based on approximations of part geometry in infinitesimal neighborhoods and part motion of an infinitesimal length. However, these first order approximations of mobility did not always predict immobility correctly. Rimon and Burdick [Rimon96] give rigorous definitions of first and second order immobility. They express any possible path

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of the part in Free Space as a series of configurations $q$ as a function of a parameter $t$. They then study the distances $d_i$ from the $i^{th}$ $C$-obstacle surface to $q(t)$. The distance is expressed as a Taylor expansion in terms of $t$ and the expansion is truncated at the first and second order terms for first and second order immobility. First order tests do not show if the distance is increasing or decreasing, as the derivative at $t = 0$ is 0.

Rimon and Blake [Rimon99] give a method to find caging grasps, configurations of jaws that constrain parts in a bounded region of C-space such that actuating the gripper results in a unique final configuration. They consider the opening parameter of the jaws as a function of the jaws’ positions along the perimeter of the part and use stratified Morse theory to find caging grasps by finding limiting cases that occur when the opening parameter is at a saddle point. [Gopalakrishnan02] also uses the distance function to determine immobile grasps of 2D polygonal parts by a pair of vertical cylindrical jaws engaging the part at its concave vertices.

Wagner [Wagner97] proposes a method to fixture rigid 3D polyhedra using struts normal to each surface. He proves that first order form closure of the polyhedron is equivalent to first order form closure of each of the three projections of the part and the contacts on to the 3 orthogonal planes. An efficient geometric algorithm to compute all placements of four frictionless point contacts on a polygonal part that ensure form closure is described by van der Stappen et al [vanderStappen99]. Given a set of four edges, they show how to compute critical contact placements in constant time. The time complexity of their algorithm is bounded by the number of such sets, their algorithm runs in an expected time of $O(n^2 \log n)$ for $n$ vertices. Cheong et al [Cheong03] give fast algorithms to find immobilizing grasps of 2D polygonal parts with 2 and 3 contacts. The algorithms find sets contact wrenches generated in wrench space that contain the center of mass.

Recent work on fixturing deformable and sheet-metal parts is based on the work of Menassa and De Vries [Menassa91] where they determine the positions of the primary datum (the datum points needed to locate the part in the correct plane) for 3-2-1 fixturing to minimize deformation. They use a finite element model of the part to model the deformation, and determine fixture locations by optimizing an objective that is a function of the deformations at the nodes. Their work is extended by [Rearick93] and [Cai96]. [Rearick93] designs a fixture for a sheet metal part by using an objective function that is a weighted sum of the norm of the deformation and the number of fixtures in the objective function. They use a remeshing algorithm, but do not address properties specific to sheet metal parts such as buckling. Cai et al describe an N-2-1 fixturing principle in [Cai96]. This is used instead of the conventional 3-2-1 principle to reduce deformation of sheet-metal parts. They use $N \geq 3$ locators for the primary datum, (i.e. they use $N$ datum points to locate the sheet metal part in the correct plane) in their fixturers. They model the sheet metal parts using finite elements with quadratic interpolation, constraining nodes in contact with the primary datum to only in-plane motion. For a known force, linear static models are used to predict deformation. To make their algorithm faster, instead of remeshing the part for different locator positions, they express the constrained displacement at the locator by using a linear interpolation of displacement at the adjacent nodes. Fixture elements are placed such that compressive forces that cause buckling do not occur.

Gopalakrishnan et al [Gopalakrishnan03, Gopalakrishnan03b] extend the work of Plut and Bone [Plut96] on holding sheet metal parts with jaws with conical grooves to non-planar deformable sheet metal parts and give a fast geometric algorithm followed by FEM iterations used in a greedy algorithm to design a fixture for the deformable part such that deformations of every FEM node conform to a specified maximum bound.

Li et al [Li02] describe a procedure to design fixtures for two sheet metal parts that are to be welded to produce a good fit along the seam to be welded. The fixturers are designed using a finite element model to determine either an optimal fixture or a robust fixture. Li et al [Li02b] describe a dexterous part holding mechanism based on vacuum cups and model the elastic deformation of the sheet-metal part using Finite Element Methods and a statistical data model. The results from this model are used to minimize the part’s deformation. Shiu et al [Shiu97, Shiu03] give a heuristic algorithm to analyze the deformation of a sheet metal part by decoupling it into beams based on the part’s features. Based on the deformations predicted, they give an algorithm to allocate tolerances to each feature. Li et al design fixturers for laser welding by first identifying a robust design space where the sensitivity of part deformations to part dimension and jaw location errors. Within this space, they use a genetic algorithm to find a fixture that minimizes an objective function defined in terms of the distance between the weld joint nodes of each weld stitch.

Path planning for elastically deformable parts has been studied using probabilistic roadmaps (PRM). Holleman et al [Holleman98] give a path planning algorithm for a flexible surface patch. They use a Bezier approximation of the surface shape and an approximate energy function to model deformation of the part. They present experimental results of paths planned for parts generated by a search graph using PRM. Guias et al [Guibas99] improve on the PRM methods for path planning for a surface patch by studying the medial axis of the workspace. Minimum energy configurations of the part are then computed for positions along the axis and connected by quasi-static paths. Lamiraux et al [Lamiraux99] generate a path for a thin rectangular elastic metal plate represented by a Bezier when it is manipulated by constraining the positions and orientations of two opposite edges. Paths are generated using PRM under elastic constraints of the part material.
3. Deformation-Space

For this paper, we use a linear FEM model (with linear interpolation, linear elasticity and no mechanical failure). We assume that we are given a 2 dimensional part $E$ discretized into a 2D triangular FEM mesh $M$. The part may or may not have holes. This FEM model of the part causes the perimeter to behave like a polygon with variable length edges, with hinges at each vertex. An initial undeformed configuration $q_0$ of the part is also specified.

Under these assumptions, we construct a Deformation Space (D-Space) $D$ of the part. Depending on the FEM model used, each of the $n$ nodes in $M$ has a predefined number $f$ of degrees of freedom. (These generally consist of translational ($r$ DOFs) and rotational/torsional ($r$ DOFs) freedoms denoted by $f_t$ and $f_r$ respectively.) Every mesh configuration is specified by specifying each of the $f$ freedoms of each of the $n$ nodes. Given the positions and geometries of all fingers, we represent the mesh in its deformation-space which is the smooth manifold $D = \mathbb{R}^r \times SO(f_t) \times SO(f_r)$ parameterized by $\Theta \in \mathbb{R}^f$ via exponential map parameterization. Thus, for a part with $n$ nodes in $M$, each with $f$ degrees of freedom each, the D-space can be parametrized by $\mathbb{R}^f$, with each coordinate axis corresponding to one DOF of one FEM node. Any deformed translated or rotated shape of this part can be represented as a unique point in this space. For any point $q$ in $D$, we denote the corresponding shape and configuration of the part by $E(q)$. Note that in our examples with triangular elements with linear elasticity and linear interpolation, $f_r = 0$.

We define $D_T$ as the set of all $q$ that preserve the topology of $M$ identical to the topology at the given undeformed configuration $q_0$. Given a finger body $A_i$, its corresponding D-space obstacle $DA_i$ is the set of all configurations $q$ such that $E(q)$ intersects $A_i$. We define $D_{free} = \bigcup_{i=1}^k DA_i^C \cap D_T$ as the set of all valid deformable configurations of $M$. $D_{free}^C$ is the complement of $D_{free}$.

Examples of configurations that lie in $D_T$ and $D_{free}^C$ are shown in Figure 2. Figure 3 shows slices of $D_{free}$ for a rectangular part with 5 nodes and a small circular finger (Figure 3(a)). Each node has 2 translational degrees of freedom about the x and y axes. Figure 3(b) assumes nodes 1-4 are fixed, and shows the resulting $D_{free}$ for node 5. Figure 3(c) assumes nodes 1, 2, 4 and 5 are fixed, and shows $D_{free}$ for node 3.

4. Potential Energy in D-Space

So far, we have looked at shapes of the part and the mesh that are geometrically possible and may be caused by an arbitrary set of finite wrenches applied on the part. We now include the mechanical properties of the part in our model simply by using the potential energy of any deformed state as predicted by the FEM model.

We assume that the part exhibits perfect linear elasticity with known Young’s modulus and Poisson’s ratio. Friction is assumed to be absent. Linear FEM analysis with linear interpolation for the given mesh and material properties is assumed to predict the part’s deformation accurately. A deformation potential energy which is a scalar value is given as a function of the deformation of each node. For the linear case, for a deformation of $X$ and a FEM stiffness matrix of $K$, the potential energy is represented as $(1/2) X^T K X$. The potential energy at a configuration $q$ in $D_{free}$ is represented as $U(q)$.

5. Deform Closure

As input, we consider: 1. the FEM mesh $M$ of the part $E$, 2. stiffness matrix $K$ of the part $E$ corresponding to mesh $M$, 3. the initial configuration $q_0$ of the mesh that specifies the initial topology and undeformed mesh shape, 4. set $A$ of frictionless contacts (finger bodies) that represents the candidate fixture, and 5. the configuration $q_f$ in which the part is held by $A$. 

![Diagram](image.png)
We propose to characterize fixtures of $E$ based on the D-Space model with potential energy. We refer to these as deform closure fixtures.

**Definition:** An equilibrium configuration is any configuration at a local minimum of the potential energy in $D_{free}$. In the absence of friction and inertial forces, any part comes to rest at an equilibrium configuration in the absence of inertial forces.

**Definition:** A set of finger bodies $A$ holding part $E$ at a configuration $q_f$ satisfies property stable ($U$) if $q_f$ is an equilibrium configuration and if any set of wrenches that increases the deformation potential energy of $E$ by at most $U$ from $U(q_f)$ is exerted and then released, the system is guaranteed to resume the initial stable configuration if inertial forces are absent. In other words, given $U \geq 0$, $A$ holds $E(q_f)$ in stable ($U$) if and only if $1$. $q_f$ is at a local minimum of the potential energy in $D_{free}$, and $2$. We consider all continuous subsets $S$ of $D_{free}$ such that for all $q_f \in S$, $U(q_f) \leq U(q_f) + U$. For all such subsets $S$, there should not exist $q_f \in S$ distinct from $q_f$ with $q_f$ at a local minimum of the potential energy in $D_{free}$.

![Potential Energy Curve](image)

*Fig. 4. For the Potential Energy curve shown, $q_A$ and $q_B$ are stable equilibria. But for the shown value of $U$, only $q_B$ satisfies stable ($U$).*

**Definition:** The threshold potential energy $U_A$ for $A$ holding $F$ in configuration $q_f$ is defined as $U_A(q_f) = \sup \{ U \mid A \text{ holding } E(q_f) \text{ satisfies stable } (U) \}$ if $q_f$ is an equilibrium configuration, and $0$ otherwise. A holds $E(q_f)$ in deform closure if and only if $U_A(q_f) > 0$.

The configuration with potential energy $U(q_f) + U_A(q_f)$ where the part can be released from the deform closure fixture is called its escape configuration. This is inspired by Rimon and Blake’s analysis of caging grasps [Rimon96]. They define a cage of a part $E$ at configuration $q_f$ by a $k$-fingered hand with opening parameter $\sigma$. A configuration $x = (q_f, \sigma)$ of the part and the hand is a cage if $q_f$ lies in a connected component of free space $C_{free}$ which is completely surrounded by the finger $C$-obstacles $CA_1, \ldots, CA_k$. They define the caging set $C$ as the set of all hand configurations that maintain the part caged between the fingers such that from any initial configuration in $C$, there exists a continuous path in $C$ leading to a desired immobilizing grasp $x_0 = (q_0, \sigma_0)$.

They use stratified Morse theory to analyze cages, and identify maximal caging grasps or puncture grasps as occurring at configurations where the distance between the fingers is at a saddle point.

Similarly in D-space, given a configuration $q_f$ that is a strict local minimum of the potential energy, we would like to identify the maximum $U$ for which $A$ holds $E(q_f)$ in deform closure, by identifying an escape configuration near $q_f$ that is a saddle point. This value of $U_A$ can be used as a metric for the deform closure fixture. An example appears in the next section.

6. **Numerical Example**

We implemented the threshold potential energy used in the definition of deform closure to a simple part with 2 triangular mesh elements and 4 nodes shown in Figure 5(a) and (b). The part is a rhombus and each element an equilateral triangle in the undeformed state. It is held by 4 point contacts, at the midpoint of each edge in Figure 5(b) near and the vertices in 5(a). The FEM model uses plane stress constant strain triangular elements made of an incompressible material (Poisson ratio=0.5). The threshold potential energies for deform closure in 5(a) and 5(b) are 4 Joules and 547 Joules respectively.

To determine this threshold configuration, we first list all limiting configurations where a vertex of the part is about to be released from the deform closure fixture. For this example, this happens only when the vertex is in contact with the jaw. Among all such configurations, we considered various cases as to each edge being in contact with or breaking contact with the corresponding jaw. We then determine the configuration with least potential energy among these. Figure 6(d) shows the escape configuration for the deform closure shown in Figure 5(a). This configuration occurs when the deformed part is still a rhombus, but one of the edges lies along 2 jaws (the ones on the left edge and the lower edge), and the opposite edge is in contact with the jaw on the right. One possible path from the configuration in Figure 5(a) to that in 5(d) is through 5(c) and 5(d) with maximum potential energy at 5(d). At no point on this path is the potential energy greater than that at the final configuration.

![Examples of Deform Closure](image)

*Fig. 5. Examples of deform closure of a rhombus shaped part with 2 triangular FEM elements. With given mesh and stiffness values, the threshold potential energies needed to release the parts from deform closure fixture (a) is 4 Joules, and 547 Joules for deform closure fixture (b). Thus fixture (b) is preferable under this model.*

If the undeformed edges of each triangle were 10mm long and the triangle part were 1mm thick and made of steel, the limiting threshold potential energy would be 547 Joules.
7. Equivalence and Frame Invariance Theorems

**Theorem 1:** If $A$ holds a rigid part in form closure, $A$ will hold the equivalent deformable part in deform closure.

By equivalent, we mean that the undeformed part has the same shape and configuration as the rigid part.

![Diagram of a part in deform closure fixture](image)

**Proof:** Since the rigid part in the given configuration is immobile, deformation occurs when the deformable part is perturbed. Since initially the deformation potential energy is at 0, the potential energy increases to above zero. Thus, $U_A > 0$. This is true for higher orders of immobility too.

**Theorem 2:** Frame Invariance.

We now show that the definition of deform closure is frame invariant, i.e. they do not depend on the global reference frame used.

The definition of $D$-space changes with the reference frame used, we prove that the definition of deform closure fixtures does not change with the global reference frame by showing the following:

(a) Displacements caused by a given force are frame invariant

(b) Potential Energy due to a given set of displacements is frame invariant, and

(c) A set of configurations of the part whose image is continuous in $D$-space for one reference frame has an image that is continuous in $D$-space for any other reference frame.

**Proof:** For mesh element $m_i$, the component of the stiffness matrix $K$ is $K_i$. Let $T_i$ be the transformation matrix between the reference frame of the element and the local reference frame, and $K_i$ is the stiffness of element $m_i$ in the local frame. $K = \sum K_i \cdot T_i = T_i \cdot K_i \cdot T_i^T$.

Let the reference frames be denoted by subscripts 1 and 2, and the transformation matrix between the frames be $T_{12}$. Thus, $T_{12} = T_i \cdot T_{i1}$.

To prove (a), we consider any given set of Forces $F_i$ expressed in reference frame 1. Thus, $X_i = K_i^{-1} F_i$. When transformed to reference frame 2, the displacement is $X_2 = T_i \cdot X_1 = \sum F_i \cdot K_i^{-1} \cdot T_{12} \cdot T_{i1} = T_{i1} \cdot K_i^{-1} \cdot T_{12} \cdot T_{i1} \cdot F_i = T_{i1} \cdot K_i^{-1} \cdot T_{12} \cdot T_{i1} \cdot F_i$ which is the displacement predicted by reference frame 2. Thus, (a) is true.

The potential energy predicted by the reference frame 1 when subject to deformations $X_1$ is $\frac{1}{2} X_1^T K_1 X_1$. The potential energy in frame 2 by the same deformation transformed to frame 2 is:

\[
\frac{1}{2} X_2^T K_2 X_2 = \frac{1}{2} \left( T_i \cdot X_1 \right)^T \left( T_{i1} \cdot K_i^{-1} \cdot T_{12} \cdot T_{i1} \right) \left( T_i \cdot X_1 \right) = \frac{1}{2} X_1^T \cdot T_{i1}^T \cdot T_{12} \cdot T_{i1} \cdot K_i^{-1} \cdot T_{i1} \cdot X_1 = \frac{1}{2} X_1^T K_1 X_1
\]

which is the same as the potential energy in frame 1. Thus, (b) is proved.

To prove (c), we show that for any 2 configurations of the part, the distance between their images in $D$-space remains the same in all frames of reference, which implies equivalence in continuity of configurations in both frames of reference.

Since $|q_a - q_b| = \sum \left( (x_{ni} - x_{nb})^2 + (y_{ni} - y_{nb})^2 \right)$ summed over all nodes $n$, where $x_{ni}$ and $y_{ni}$ are the x and y co-ordinates of node $n$ in configuration $i = a, b$. Since we consider only distance-preserving transformations when changing reference frames, $\left( (x_{ni} - x_{nb})^2 + (y_{ni} - y_{nb})^2 \right)$ is frame invariant for all nodes $n$. Thus, $|q_a - q_b|$ is also frame-invariant.

8. Discussion

8.1 Symmetry in $D$-Space

If a model of $D$-space were constructed by brute force, the amount of computation involved in determining $D_{free}$ may be extremely high. For example, for the 12 noded part in Figure 1, the $D$-Space will be 24-dimensional with linear elasticity and constant strain triangles. However, computing $D_{free}$ will be simplified to a large extent by the symmetries of $D$-obstacles and of $D_T$.

$D$-obstacles are symmetric, since for any two triangles of the mesh, the collision check for any finger body is identical. Hence, for every triangle in the mesh, the slice of the $D$-obstacle with fixed positions of other nodes will look identical. Further, the $D$-obstacle is defined by only the union of these slices, since if any portion of the part collides with the obstacle, at least one triangle in the mesh will collide with it. For the part in Figure 1, this causes the $D$-obstacles to have a 12-fold symmetry due to presence of 12 elements in the mesh. Further, for identical obstacles, each $D$-obstacle is a copy of every other obstacle displaced by a known vector.
Although the set of topology preserving mesh configurations $D_T$ may not be symmetric in itself, we can consider a superset $D'_T$ of $D_T$ that consists of all meshes where no triangles intersect and no triangle has zero area. $D'_T$ contains many configurations that cannot be attained by the physical part (such as a mirror image of the undeformed mesh), but there is never a continuous path in $D'_T$ connecting the initial mesh to an unattainable mesh. For the part in Figure 1, this results in 23 symmetries because of pairs of distinct triangles, 29 symmetries for pairs of triangles with 2 distinct vertices each, and 13 symmetries for pairs of triangles that have 1 distinct vertex each.

8.2 Conclusion

We have presented a framework for characterizing fixtures of deformable parts. We define fingers $A$ to hold part $E$ in deform closure if the increase in potential energy needed to release $E$ from $A$ is non-zero. The minimum amount of work needed to release $E$ is used as a metric for the deform closure fixture. We present a numerical example for this.

The next step is to develop algorithms that apply this framework to compute deform closure fixtures in 2D and 3D for solid and sheet metal parts.

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